Bits, Bytes and Integers

Introduction to Computer Systems

https://xjtu-ics.github.io/

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Today: Bits, Bytes, and Integers

- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, multiplication, shifting
- **Representations in memory, pointers, strings**

Everything is bits

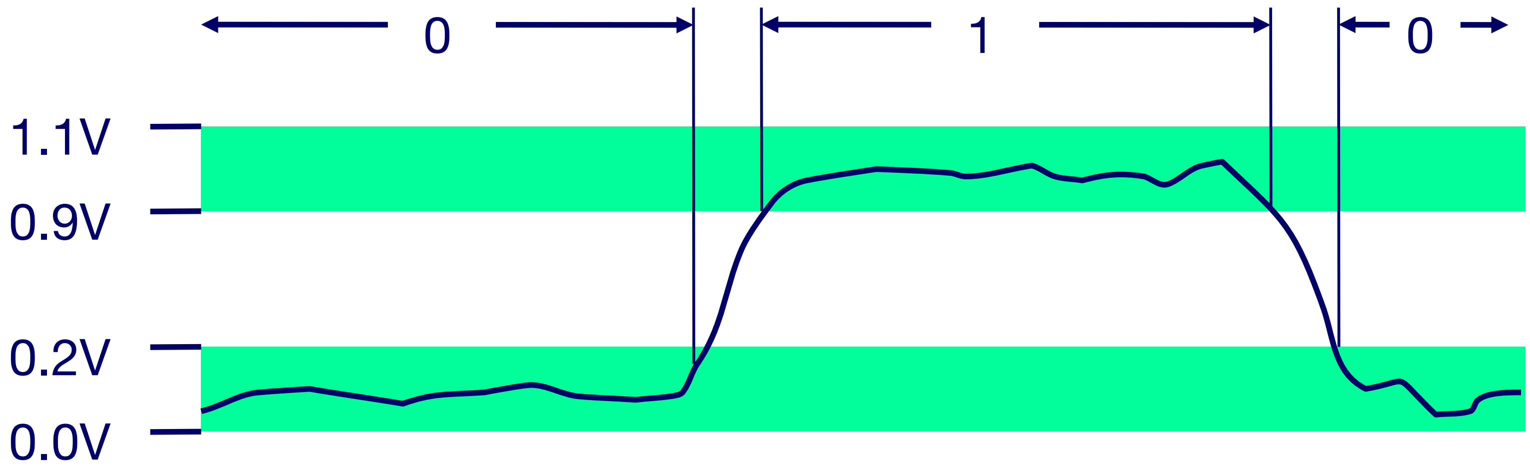
Each bit is 0 or 1

By encoding/interpreting sets of bits in various ways

- Computers determine what to do (instructions)
- … and represent and manipulate numbers, sets, strings, etc…

Why bits? Electronic Implementation

- Easy to store with bistable elements (双稳态器件)
- Reliably transmitted on noisy and inaccurate wires



For example, can count in binary

Base 2 Number Representation

Represent 15213_{10} as 11101101101101_2

Represent 1.20_{10} as $1.0011001100110011[0011]\ldots_2$

Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$

Encoding Byte Values

Byte = 8 bits

- Binary 00000000_2 to 11111111_2
- Decimal: 0_{10} to 255_{10}
- Hexadecimal 00_{16} to FF_{16}
  - Base 16 number representation
  - Use characters '0' to '9' and 'A' to 'F'
  - Write $FA1D37B_{16}$ in C as
    - 0xFA1D37B
    - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

$$15213:\ \underbrace{0011}_{3}\ \underbrace{1011}_{B}\ \underbrace{0110}_{6}\ \underbrace{1101}_{D}$$

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8
`float`	4	4
`double`	8	8
pointer	4	8

Example Data Representations

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`float`	4	4
`double`	8	8
pointer	4	8
	"ILP32"	"LP64"

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Boolean Algebra

Developed by George Boole in 19th Century

Algebraic representation of logic

Encode "True" as 1 and "False" as 0

And

A&B = 1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

Or

A|B = 1 when either A=1 or B=1 or both

|	0	1
0	0	1
1	1	1

Not

~A = 1 when A=0

~	0	1
	1	0

Exclusive-Or (Xor)

A^B = 1 when A=1 or B=1, but not both

^	0	1
0	0	1
1	1	0

General Boolean Algebras

Operate on Bit Vectors

Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

Bit-Level Operations in C

Operations &, |, ~, ^ Available in C

- Apply to any “integral” data type
  - `long, int, short, char, unsigned`
- View arguments as bit vectors
- Arguments applied bit-wise

Contrast: Logic Operations in C

Contrast to Bit-Level Operators

Logic Operations: &&, ||, !

- View 0 as "False"
- Anything nonzero as "True"
- Always return 0 or 1
- Early termination

Examples (char data type)

```
!0x41 →   0x00
!0x00 →   0x01
!!0x41→   0x01

0x69 && 0x55 →   0x01
0x69 || 0x55 →   0x01
p && *p
```

(avoids null pointer access)

Watch out for && vs. & (and || vs. |)…
Super common C programming pitfall!

Shift Operations

Left Shift: `x << y`

Shift bit-vector **x** left **y** positions

– Throw away extra bits on left

Fill with 0's on right

Right Shift: `x >> y`

Shift bit-vector **x** right **y** positions

Throw away extra bits on right

Logical shift

Fill with 0's on left

Arithmetic shift

Replicate most significant bit on left

Undefined Behavior

Shift amount < 0 or ≥ word size

Argument **x**	01100010
<< 3	00010000
Log. >> 2	00011000
Arith. >> 2	00011000

Argument **x**	10100010
<< 3	00010000
Log. >> 2	00101000
Arith. >> 2	11101000

Today: Bits, Bytes, and Integers

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Question?

```
int foo = -1;
unsigned bar = 1;

(foo < bar) == true ?
```

Encoding "Integers"

Unsigned

Signed (twos complement)

Given a bit vector x, w bits long…

$$\mathrm{B2U}(x) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

$$\mathrm{B2T}(x) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

Examples (w = 5)

±16	8	4	2	1
0	1	0	1	0

$0 + 8 + 0 + 2 + 0 = \quad 10$

16	8	4	2	1
1	0	1	1	0
-16	8	4	2	1

$16 + 0 + 4 + 2 + 0 = \quad 22$

$-16 + 0 + 4 + 2 + 0 = -10$

Negation: Complement & Increment

Negate through complement and increase

```
~x + 1 == -x
```

Why?

```
-x + x       == 0  (by definition)
~x + x       == 1111…111 == -1
~x + x + 1 == 0
(~x+1) + x == 0
 ~x+1        == -x
```

x	1	0	0	1	1	1	0	1
+ ~x	0	1	1	0	0	0	1	0
−1	1	1	1	1	1	1	1	1

Example: x = 15213

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
~x	-15214	C4 92	11000100 10010010
~x+1	-15213	C4 93	11000100 10010011
y	-15213	C4 93	11000100 10010011

Complement & Increment Examples

$x = 0$

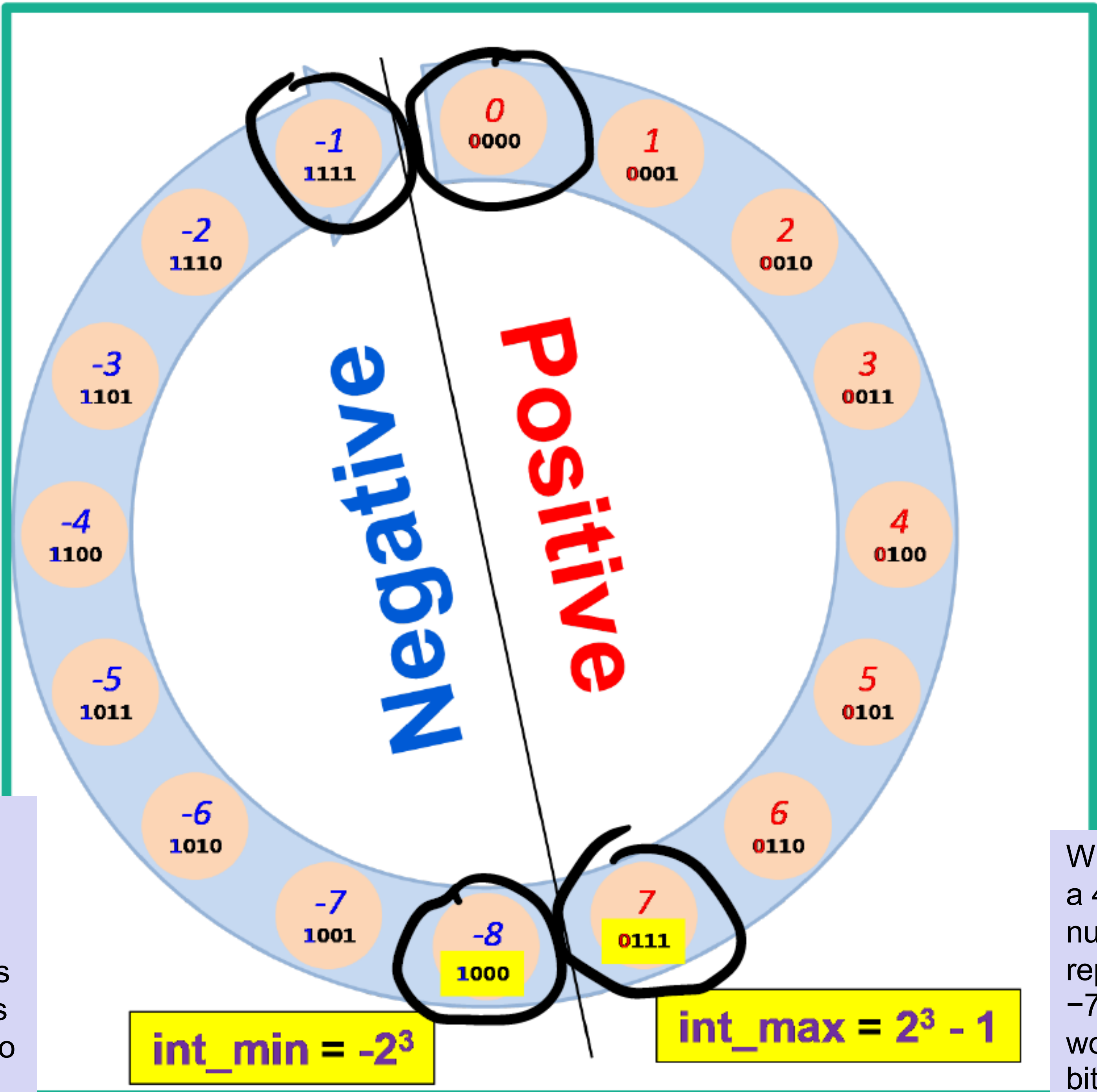
	Decimal	Hex	Binary
0	**0**	00 00	00000000 00000000
~0	**-1**	FF FF	11111111 11111111
~0+1	**0**	00 00	00000000 00000000

$x = T_{\min}$

	Decimal	Hex	Binary
x	**-32768**	80 00	10000000 00000000
~x	**32767**	7F FF	01111111 11111111
~x+1	**-32768**	80 00	10000000 00000000

Oops!
It's still
negative!

Eight negative values:
−1, − 2, …, − 8



Eight *non*-negative values:
0, 1, …, 7

Mathematicians would prefer it if a 4-bit signed number could represent values −8…8, but that's $2^4 + 1$ values, so they won't all fit.

What if we made a 4-bit signed number only represent values −7…7? Then we wouldn't be using bit pattern `1000`…

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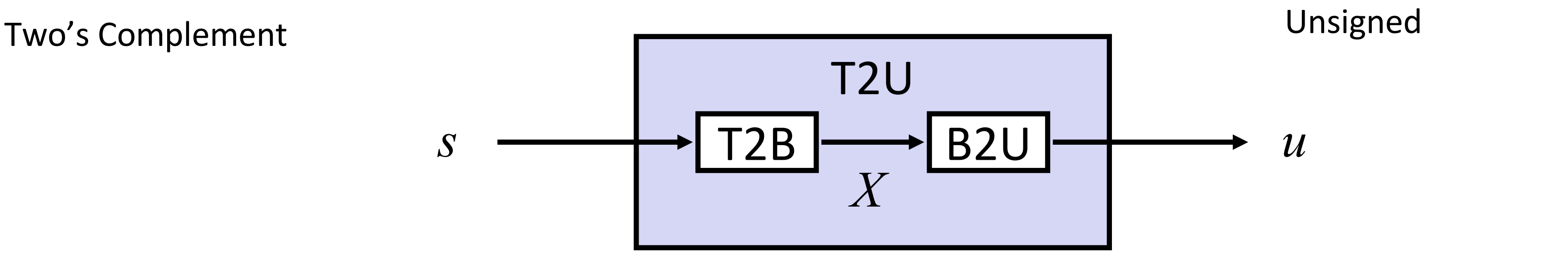
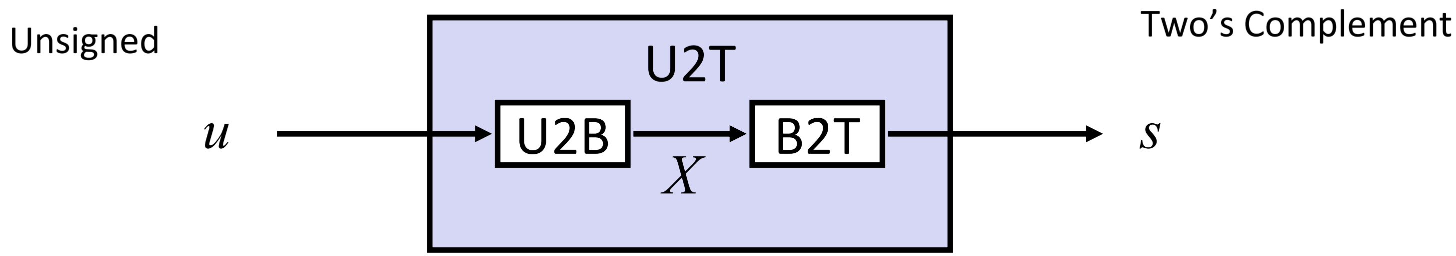
Mapping Between Signed & Unsigned



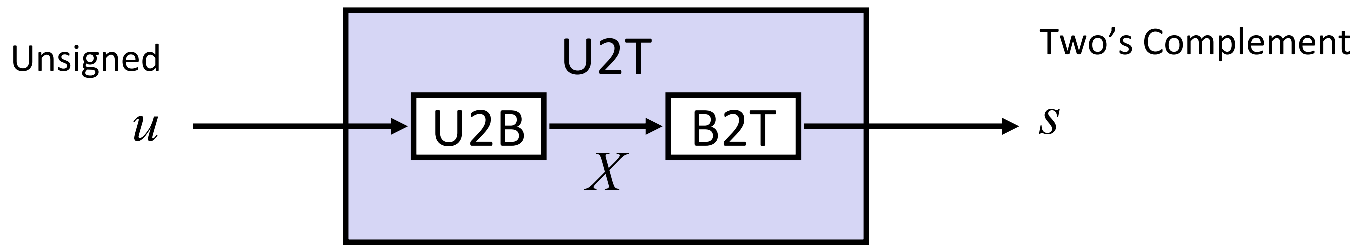
Maintain Same Bit Pattern



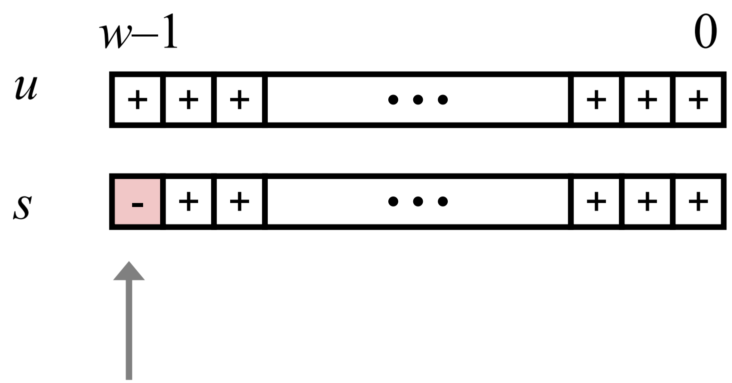
Maintain Same Bit Pattern

Mappings between unsigned and two's complement numbers:
Keep bit representations and reinterpret

Relation between Signed & Unsigned



Maintain Same Bit Pattern



Large positive weight
becomes
Large negative weight

Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

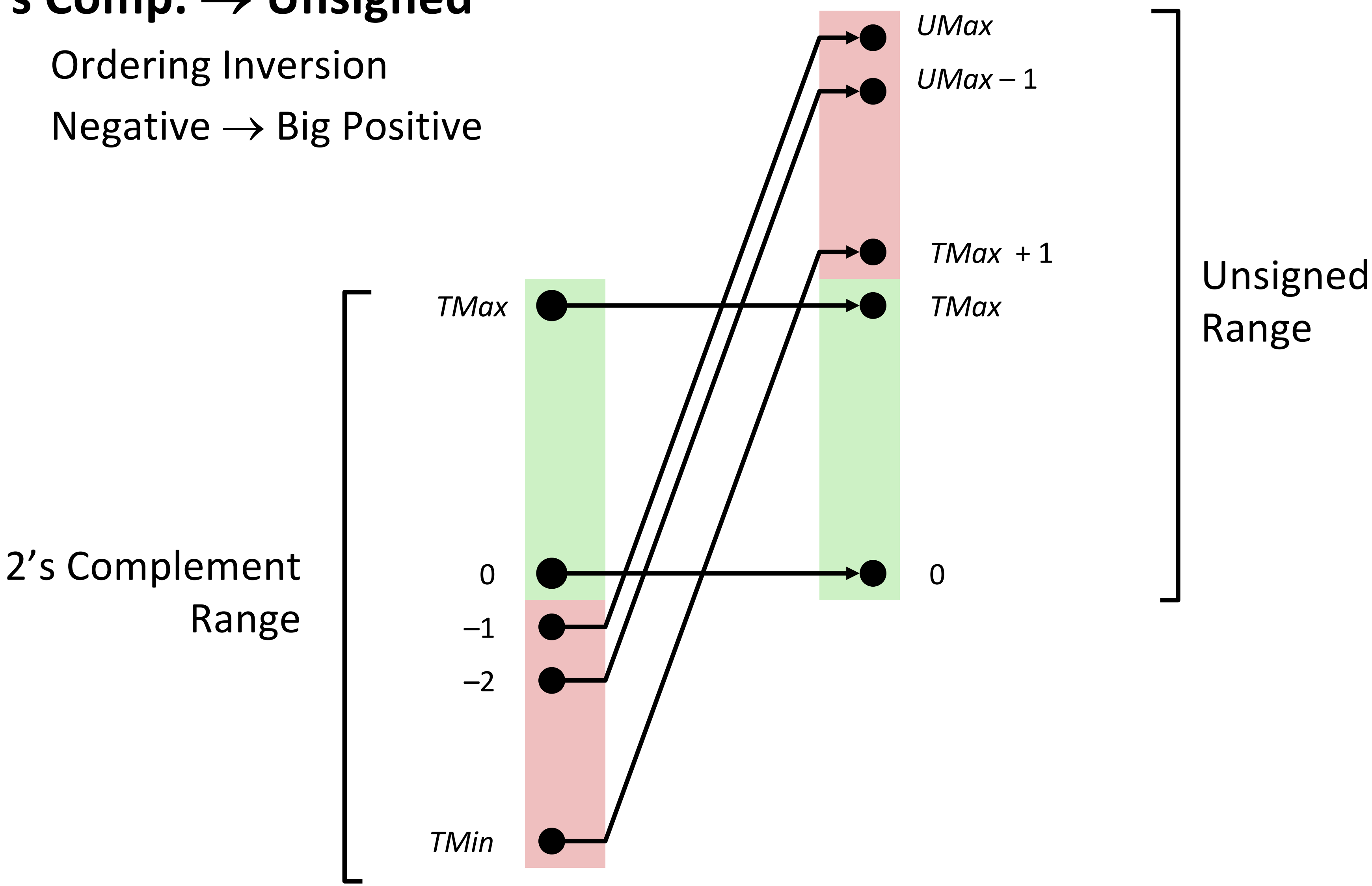
$=$

± 16

Conversion Visualized

2's Comp. → Unsigned

- Ordering Inversion
- Negative → Big Positive



Signed vs. Unsigned in C

Constants

By default are considered to be signed integers

Unsigned if have "U" as suffix

`0U, 4294967259U`

Casting

Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

Implicit casting also occurs via assignments and procedure calls

```
tx = ux;                    int fun(unsigned u);
uy = ty;                    uy = fun(tx);
```

Question?

example02.c

```
int foo = -1;
unsigned bar = 1;

foo < bar == true ?
```

Casting Surprises

Expression Evaluation

If there is a mix of unsigned and signed in single expression,
signed values implicitly cast to unsigned

Including comparison operations **<, >, ==, <=, >=**

Examples:

Constant 1	Constant 2	Relation	Evaluation
`0`	`0U`	==	Unsigned
`-1`	`0`	<	Signed
`-1`	`0U`	>	Unsigned
`INT_MAX`	`INT_MIN`	>	Signed
`(unsigned)INT_MAX`	`INT_MIN`	<	Unsigned
`-1`	`-2`	>	Signed
`(unsigned)-1`	`-2`	>	Unsigned
`INT_MAX`	`((unsigned)INT_MAX) + 1`	<	Unsigned
`INT_MAX`	`(int)(((unsigned)INT_MAX) + 1)`	>	Signed

Summary
Casting Signed ↔ Unsigned: Basic Rules

- **Bit pattern is maintained**
- **But reinterpreted**
- **Can have unexpected effects: adding or subtracting 2^w**

Expression containing signed and unsigned int

- `int` is cast to `unsigned`!!

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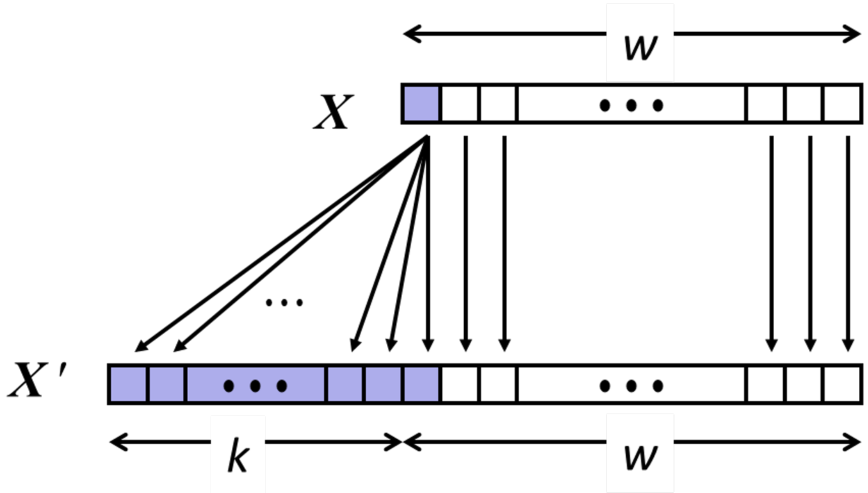
Question?

example03.c

```
int x = 0x8000;
short sx = (short) x;
int y = sx;
```

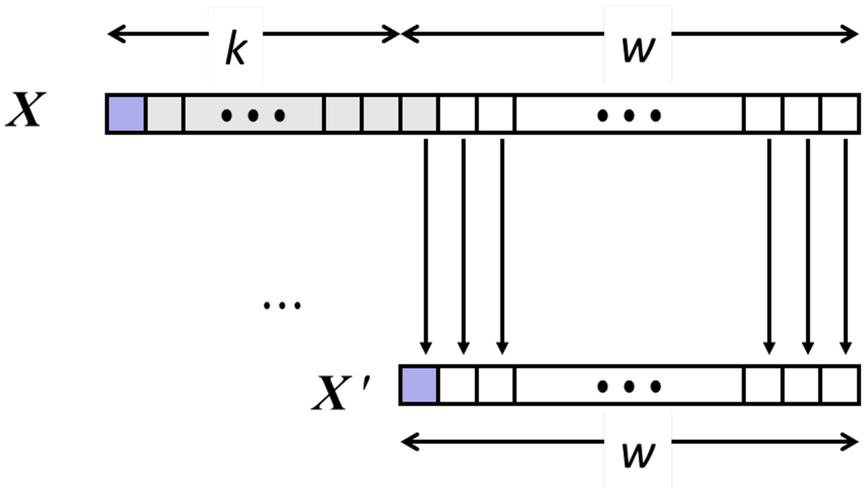
Sign Extension and Truncation

Sign Extension



Make $\boldsymbol{\kappa}$ copies of sign bit

Truncation



Chop off $\boldsymbol{\kappa}$ highest bits

Sign Extension: Simple Example

Positive number

	-16	8	4	2	1
10 =	0	1	0	1	0

	-32	16	8	4	2	1
10 =	0	0	1	0	1	0

Negative number

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-32	16	8	4	2	1
-10 =	1	1	0	1	1	0

Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

Question?

example03.c

```
int x = 0x8000;
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```

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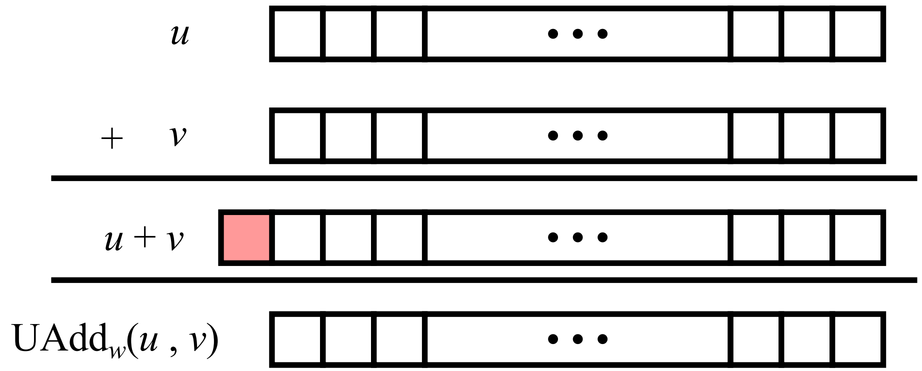
Unsigned Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



Standard Addition Function

Ignores carry output

```
unsigned char     1110 1001      E9       233
                + 1101 0101    + D5     + 213
```

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

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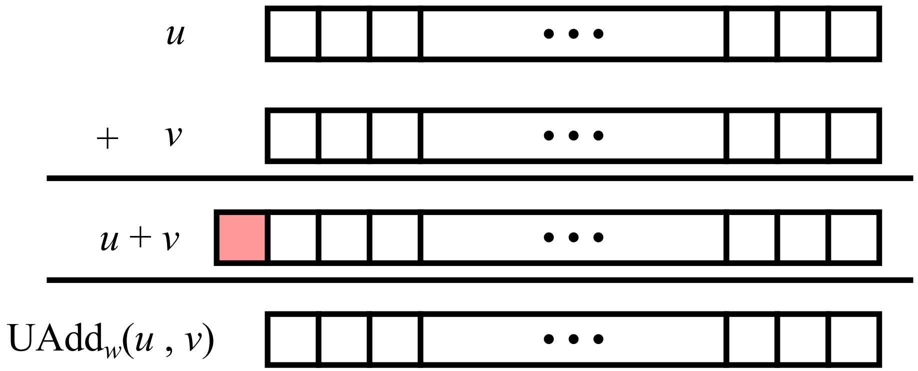

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Hex	Decimal	Binary
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8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

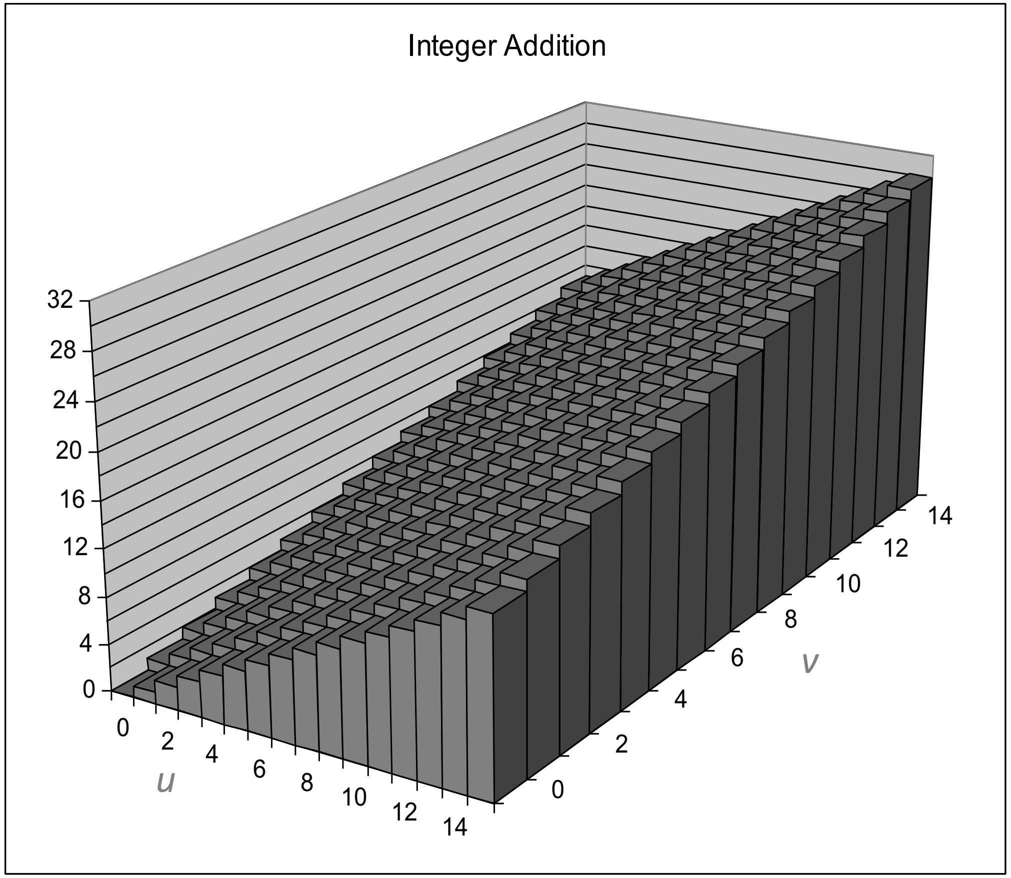
```
unsigned char      1110 1001      E9      233
                 + 1101 0101    + D5    + 213
                 -----------    ----    -----
                 1 1011 1110     1BE      446
                 -----------    ----    -----
                   1011 1110      BE      190
```

Visualizing (Mathematical) Integer Addition

Integer Addition

- 4-bit integers *u*, *v*
- Compute true sum $Add_4(u, v)$
- Values increase linearly with *u* and *v*
- Forms planar surface

$Add_4(u, v)$

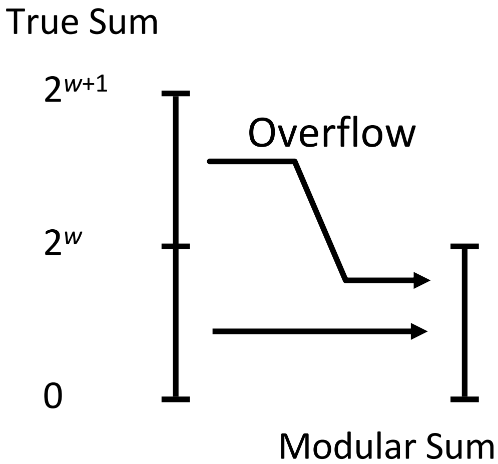


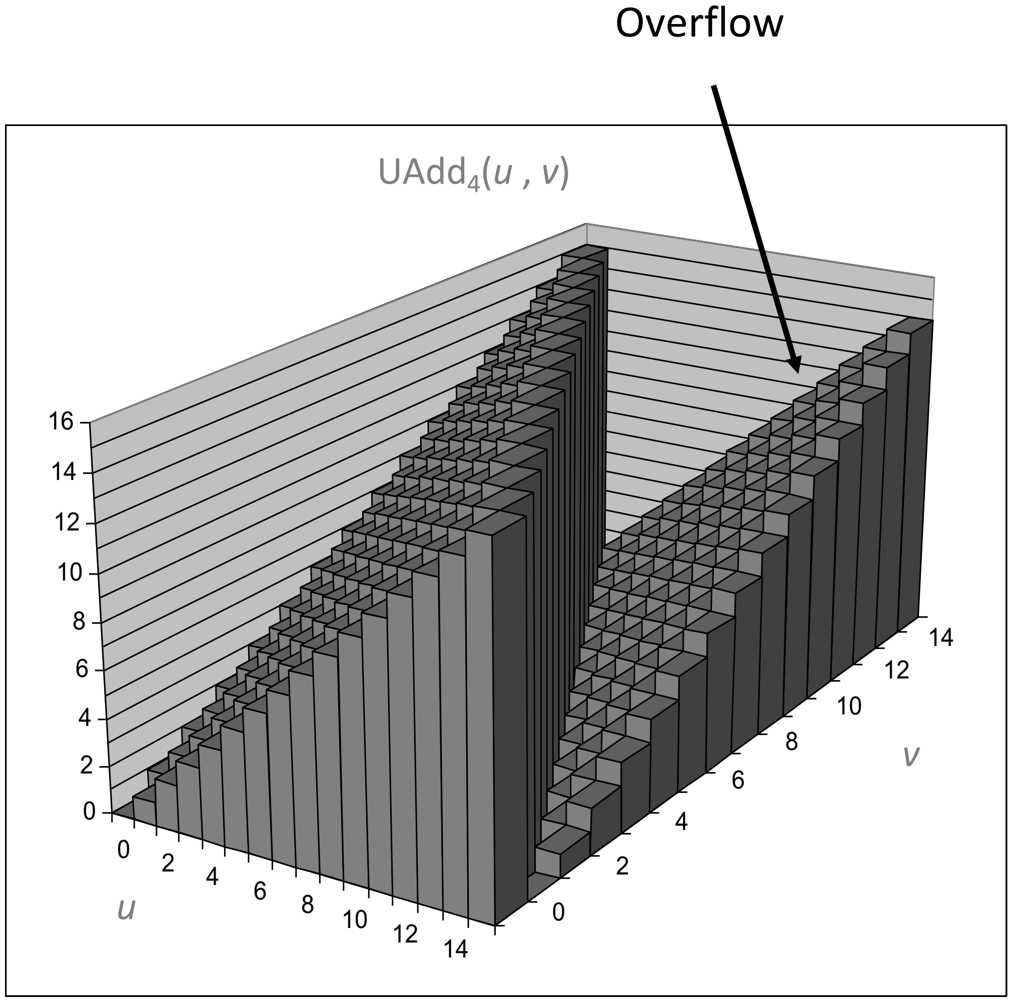
Visualizing Unsigned Addition

Wraps Around

If true sum ≥ 2^w

At most once



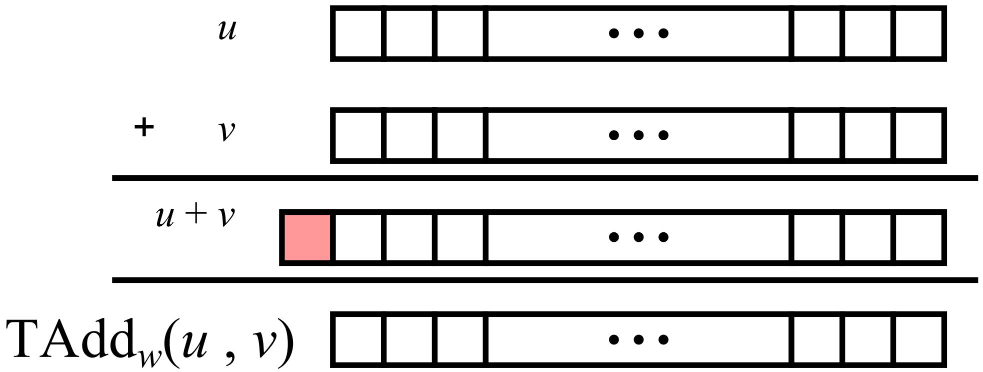


Two's Complement Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



TAdd and UAdd have Identical Bit-Level Behavior

Signed vs. unsigned addition in C:

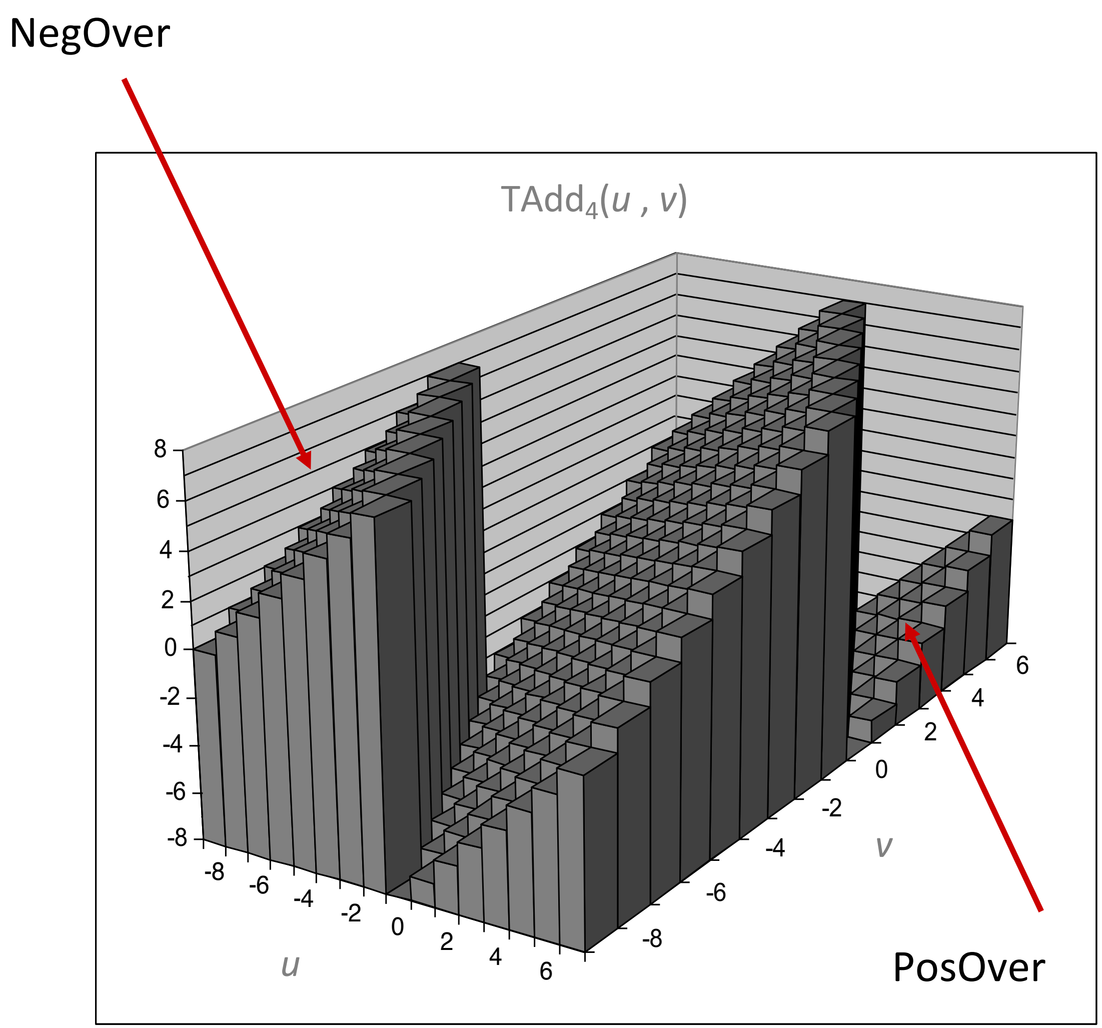
```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

Will give `s == t`

1110 1001	E9	-23
+ 1101 0101	+ D5	+ -43
1 1011 1110	1BE	-66
1011 1110	BE	-66

Visualizing 2's Complement Addition

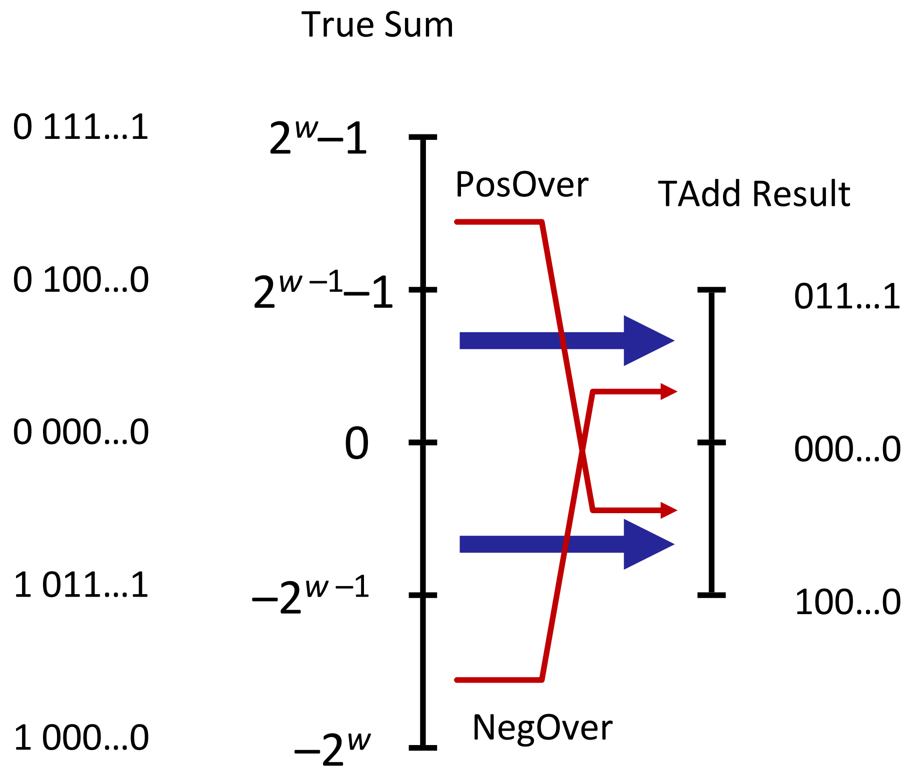
- **Values**
  - 4-bit two's comp.
  - Range from -8 to +7
- **Wraps Around**
  - If sum $\geq 2^{w-1}$
    - Becomes negative
    - At most once
  - If sum $< -2^{w-1}$
    - Becomes positive
    - At most once



TAdd Overflow

Functionality

- True sum requires w+1 bits
- Drop off MSB
- Treat remaining bits as 2's comp. integer



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Shifting

- **Left Shift: `x << y`**
  - Shift bit-vector `x` left `y` positions
  - Throw away extra bits on left
  - Fill with 0's on right
  - Equivalent to multiplying by 2^y
- **Right Shift: `x >> y`**
  - Shift bit-vector `x` right `y` positions
  - Throw away extra bits on right
  - Two kinds:
    - "Logical": Fill with 0's on left
    - "Arithmetic": Replicate most significant bit on left
  - *Almost* equivalent to dividing by 2^y
- **Undefined Behavior (in C)**
  - Shift amount < 0 or ≥ word size

Argument x	01100010
<< 3	00010000
Logical >> 2	00011000
Arithmetic >> 2	00011000

Argument x	10100010
<< 3	00010000
Logical >> 2	00101000
Arithmetic >> 2	11101000

Multiplication

Goal: Computing Product of *w*-bit numbers *x*, *y*

Either signed or unsigned

But, exact results can be bigger than *w* bits

Unsigned: up to $2w$ bits

Result range: $0 \leq x * y \leq (2^w - 1)^2 = 2^{2w} - 2^{w+1} + 1$

Two's complement min (negative): Up to $2w$-1 bits

Result range: $x * y \geq (-2^{w-1})*(2^{w-1}-1) = -2^{2w-2} + 2^{w-1}$

Two's complement max (positive): Up to $2w$ bits, but only for $(TMin_w)^2$

Result range: $x * y \leq (-2^{w-1})^2 = 2^{2w-2}$

So, maintaining exact results…

would need to keep expanding word size with each product computed

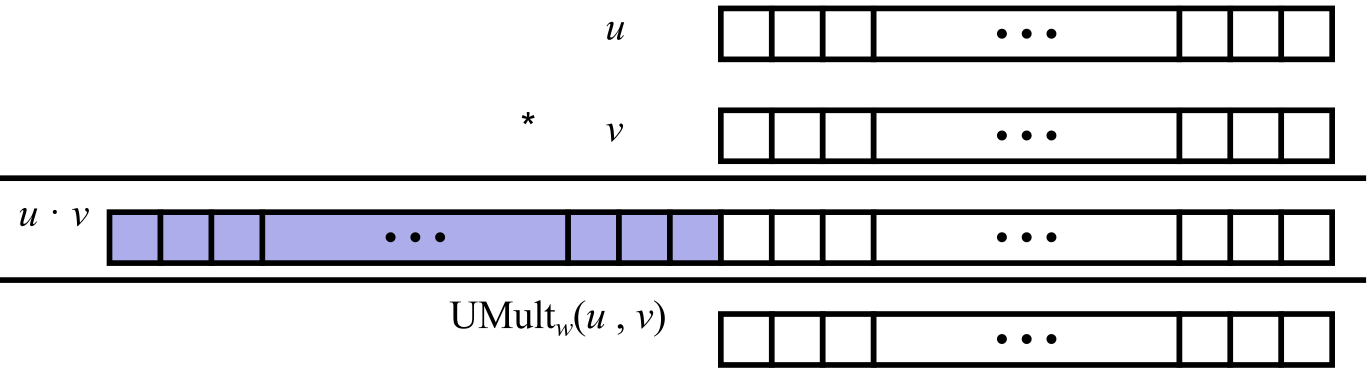
is done in software, if needed

Unsigned Multiplication in C

Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits



Standard Multiplication Function

Ignores high order *w* bits

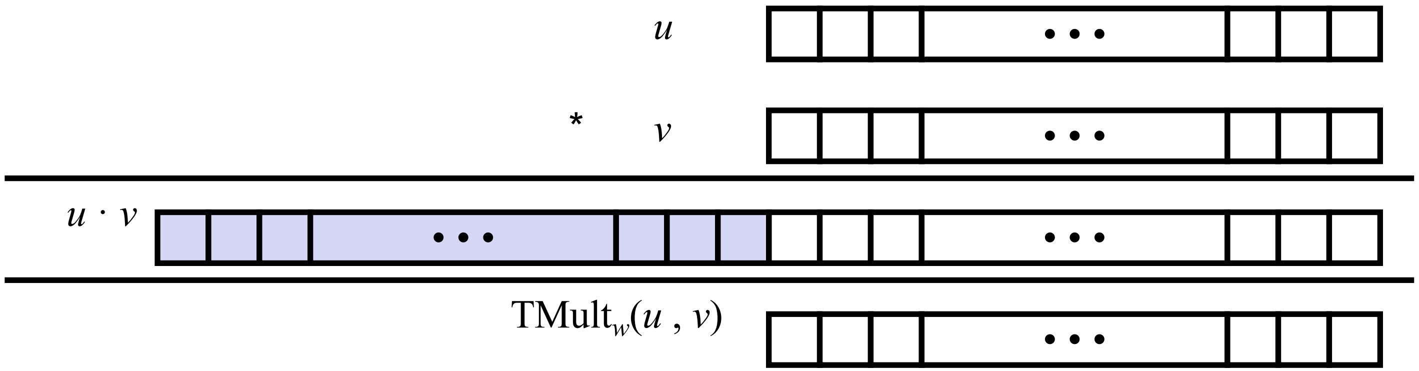
```
                    1110 1001        E9        233
*                   1101 0101    *   D5    *   213
---------------------------------------------------
  1100 0001 1101 1101             C1DD       49629
---------------------------------------------------
                    1101 1101        DD        221
```

Signed Multiplication in C

Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits



Standard Multiplication Function

Ignores high order *w* bits

Some of which are different for signed vs. unsigned multiplication

Lower bits are the same

1110 1001	E9	-23
* 1101 0101	* D5	* -43
0000 0011 1101 1101	03DD	989
1101 1101	DD	-35

Power-of-2 Multiply with Shift

Operation

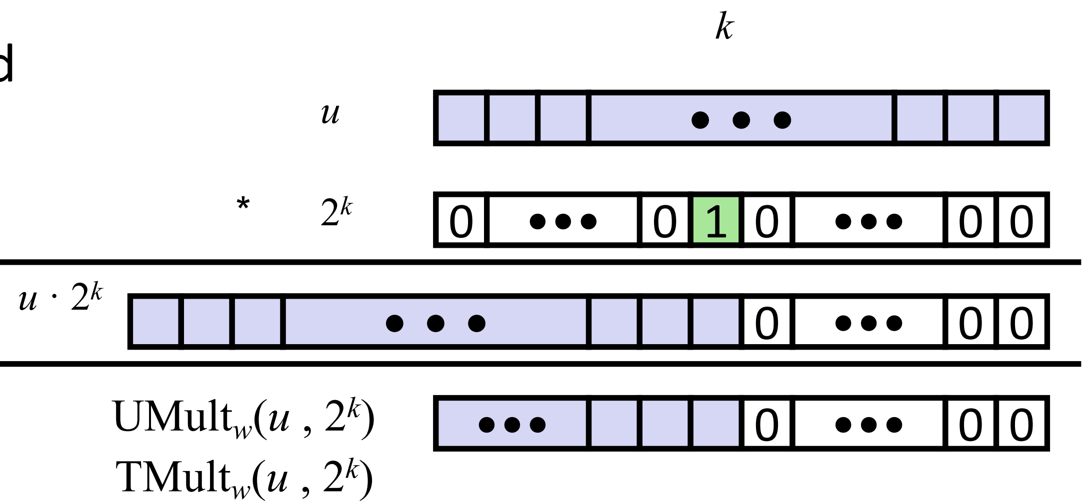
`u << k` gives `u * 2`^{k}

Both signed and unsigned

Operands: *w* bits

True Product: *w*+*k* bits

Discard *k* bits: *w* bits



Examples

```
u << 3          ==   u * 8
(u << 5) - (u << 3)      ==     u * 24
```

Most machines shift and add faster than multiply

Compiler generates this code automatically

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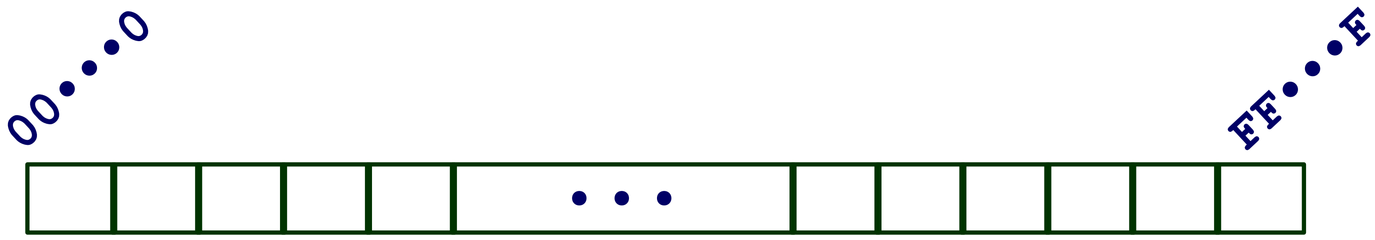
Byte-Oriented Memory Organization



Programs refer to data by address

- Imagine all of RAM as an enormous array of bytes
- An address is an index into that array
  - A pointer variable stores an address

System provides a private *address space* to each "process"

- A process is an instance of a program, being executed
- An address space is one of those enormous arrays of bytes
- Each program can see only its own code and data within its enormous array
- We'll come back to this later ("virtual memory" classes)

Machine Words

Any given computer has a "Word Size"

- Nominal size of integer-valued data
  - and of addresses

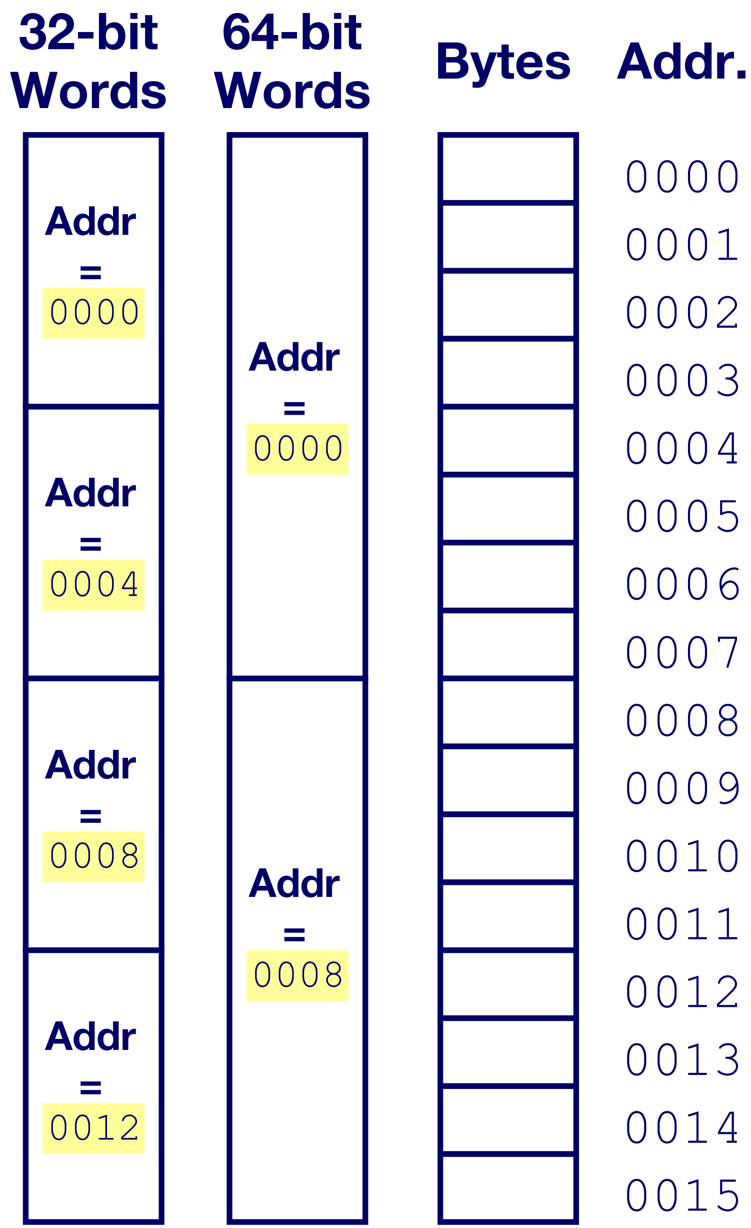
- Historically, most machines used 32 bits (4 bytes) as word size
  - Limits addresses to 4GB (2^{32} bytes)

- Currently, machines have 64-bit word size
  - Potentially, could have 16 EB (exabytes) of addressable memory
  - That's 18.4×10^{18} bytes

- Machines still support multiple data formats
  - Fractions or multiples of word size
  - Always integral number of bytes

Addresses *Always* Specify Byte Locations

- Address of a word is address of the first byte in the word
- Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
`char`	1	1	1
`short`	2	2	2
`int`	4	4	4
`long`	4	8	8
`float`	4	4	4
`double`	8	8	8
pointer	4	8	8

Byte Ordering

So, how are the bytes within a multi-byte word ordered in memory?

Conventions

- Big Endian: Sun, PPC Mac, *network packet headers*
  - Least significant byte has highest address
- Little Endian: *x86*, ARM processors running Android, iOS, and Windows
  - Least significant byte has lowest address

Byte Ordering Example

Example

Variable x has 4-byte value of 0x01234567

Address given by &x is 0x100

Big Endian

		0x100	0x101	0x102	0x103		
		01	23	45	67		

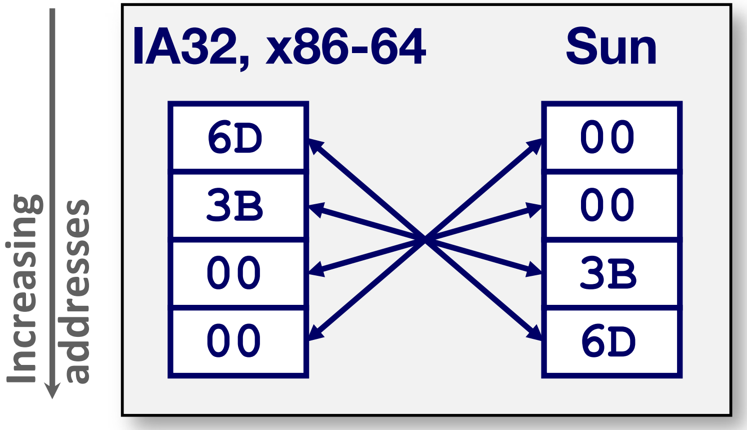
Little Endian

		0x100	0x101	0x102	0x103		
		67	45	23	01		

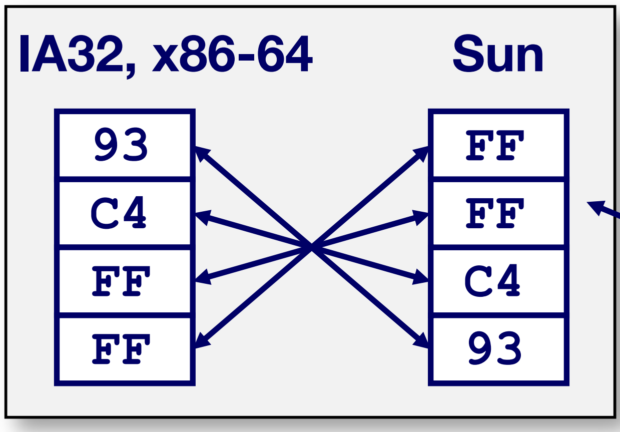
Representing Integers

Decimal:	`15213`			
Binary:	`0011`	`1011`	`0110`	`1101`
Hex:	`3`	`B`	`6`	`D`

`int A = 15213;`

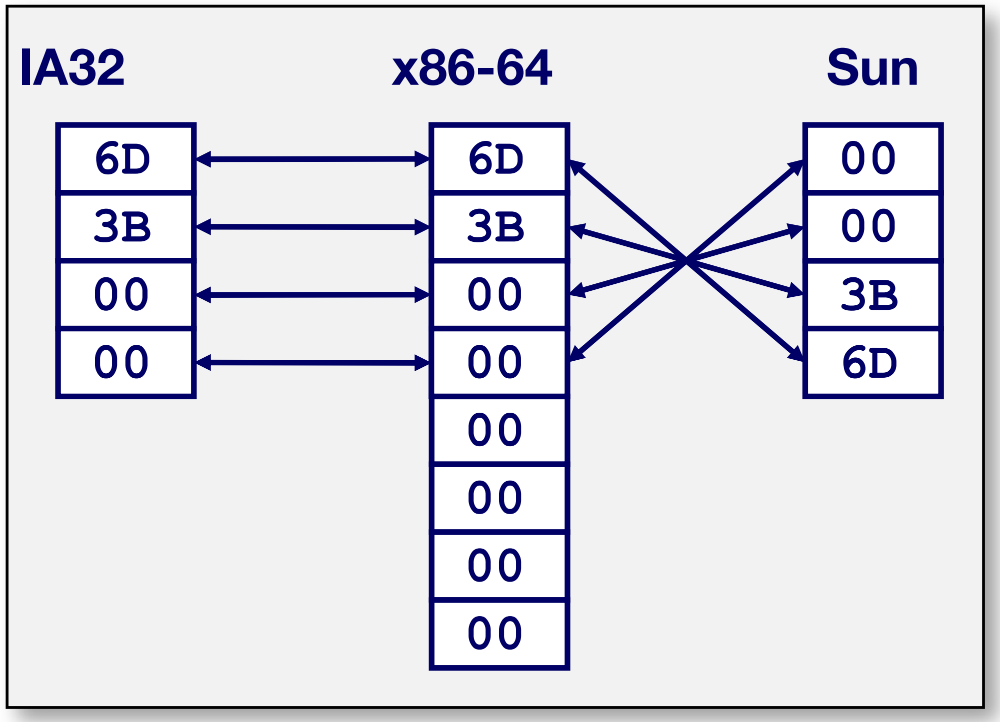


`int B = -15213;`



Two's complement representation

`long int C = 15213;`



Examining Data Representations

Code to Print Byte Representation of Data

Casting pointer to unsigned char * allows treatment as a byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, size_t len){
  size_t i;
  for (i = 0; i < len; i++)
    printf(”%p\t0x%.2x\n",start+i, start[i]);
  printf("\n");
}
```

Printf directives:

%p: Print pointer

%x: Print Hexadecimal

show_bytes Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

Result (Linux x86-64):

```
int a = 15213;
0x7fffb7f71dbc    6d
0x7fffb7f71dbd    3b
0x7fffb7f71dbe    00
0x7fffb7f71dbf    00
```

Representing Pointers

```
int B = -15213;
int *P = &B;
```

Sun	IA32	x86-64
EF	AC	3C
FF	28	1B
FB	F5	FE
2C	FF	82
		FD
		7F
		00
		00

Different compilers & machines assign different locations to objects

Even get different results each time run program

Representing Strings

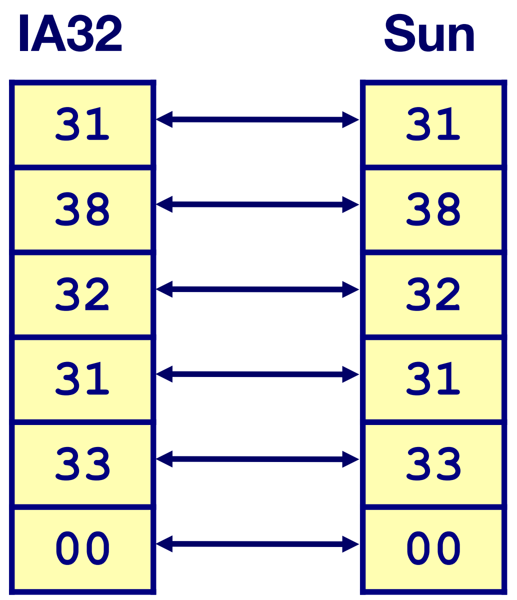
```
char S[6] = "18213";
```

Strings in C

- Represented by array of characters
- Each character encoded in ASCII format
  - Standard 7-bit encoding of character set
  - Character "0" has code 0x30
    - Digit *i* has code 0x30+*i*
- String should be null-terminated
  - Final character = 0

Compatibility

- Byte ordering not an issue



Representing x86 machine code

x86 machine code is a sequence of *bytes*

Grouped into variable-length instructions, which look like strings…

But they contain embedded little-endian numbers…

Example Fragment

Address	Instruction Code	Assembly Rendition	
`8048365:`	`5b`	`pop`	`%ebx`
`8048366:`	`81 c3 ab 12 00 00`	`add`	`$0x12ab,%ebx`
`804836c:`	`83 bb 28 00 00 00 00`	`cmpl`	`$0x0,0x28(%ebx)`

Deciphering Numbers

Value:	`0x12ab`
Pad to 32 bits:	`0x000012ab`
Split into bytes:	`00 00 12 ab`
Reverse:	`ab 12 00 00`